Single-input Multiple-output Multi-carrier Wireless Indoor Direction Finding in a Compact Multipath Scenario Using 2.4 GHz ISM Band2

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Abstract— Super-resolution direction of arrival (DoA) estimation of impinging signals from a line of sight (LOS) source and multiple point scatterers at the 2.4 GHz ISM band is considered within a compact area. The channel is partially frequency flat in the interested range and all the impinging signals due to the point scatterers are correlated, which introduces the necessity of algorithms for decorrelation of signals. This paper introduces a frequency domain decorrelation scheme for the DoA estimation of a LOS source and point scatterers, which constraints a particular structure on the receiver antenna. The limitation in terms of resolution is demonstrated with respect to the coherence bandwidth of the channel for distinct cases of point scatterer locations with respect to the LOS source.

1. INTRODUCTION

The high-resolution localization of impinging signals originating from a wireless node using the 2.4 GHz band offers a wide range of applications [1, 2]. A case of interest is the positioning of sources in compact environments which suffer from multipath scattering. For such a problem, a super-resolution direction of arrival (DoA) estimation of the line-of-sight (LOS) signal and the scatterers is considered.

There is an abundancy of research on the estimation problem of DoA. For achieving super resolution of DoA estimation, subspace based estimators such as MUSIC are promoted [3]. The estimators try to separate distinct impinging signals assuming that they are at least uncorrelated to some extent. However, for environments that introduce multipath scattering the impinging signals are highly correlated. It is well known that subspace based estimators are significantly fallacious when the impinging signals are correlated [4].

This work proposes a multi-carrier frequency domain decorrelation scheme which imposes a multiple gain pattern structure on the receiver antenna for using a super-resolution subspace based estimator in environments where multipath scattering exists and the impinging signals are correlated. In a first step, we demonstrate the signal model, the scenario and we formulate the correlated source problem referring to the coherence bandwidth of the channel. Then, the receiver concept of a multi-beam single-input multiple-output (SIMO) system for the 2.4 GHz ISM band follows, together with simulated DoA estimation results. The receiver concept proposes a frequency domain algorithm that decorrelates the impinging signals to an extent that is dependent on the coherence bandwidth of the channel; we demonstrate the limitations for the resolution of the estimation.

2. SIGNAL MODEL AND PROBLEM FORMULATION

The model in this work assumes that there exist a LOS source and multiple point scatterers, which reflect the signal that is transmitted by the LOS source. A carrier-only signal is transmitted and received, and it is sequentially swept covering the whole bandwidth of 80 MHz starting at 2.4 GHz with 1 MHz steps. These frequencies are indexed as f_k where $k = 1, 2, \ldots, K = 80$. The LOS source and the scatterers are referred as $s_{n,k}$ for $n = 1, 2, \ldots, N$. The LOS source corresponds to $n = 1$ and there exist $N - 1$ point scatterers, which correspond to $n > 1$.

For signal reception, there is a receiver element which provides different gain patterns $g_l(\theta)$ for $l = 1, 2, \ldots, L$. Thus, the signal that is received at the gain pattern $g_l(\theta)$ at the frequency k is represented as $r_{l,k}$. The location of the LOS source and the scatterers with respect to the reference point of the receiver, are indicated by the position vector $\vec{s_n}$ $\vec{e_i}$ respectively in Figure 1.

The complex signal that is transmitted by the LOS source frequency f_k is $s_{1,k}(t) = \alpha_k e^{j2\pi f_k t}$ where $\alpha_k \in \mathbb{C}$ represents its amplitude and phase at f_k. Taking the propagation time τ_n from the

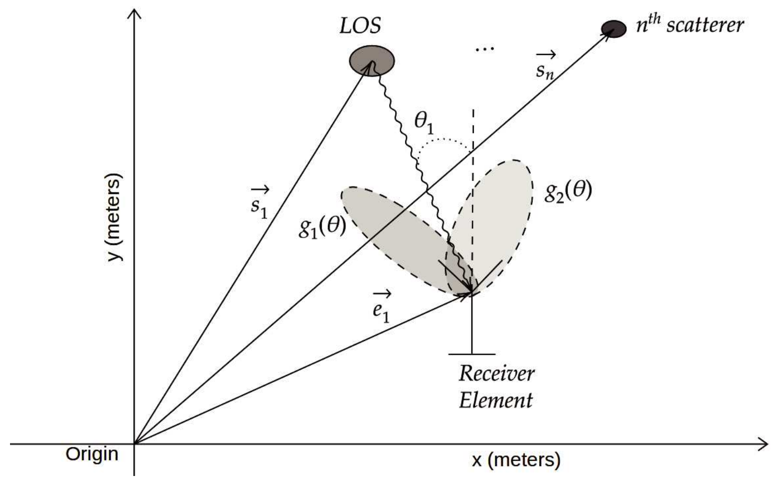

Figure 1: A demonstration of notation that is occupied in the paper.

LOS source to the scatterer n and the complex attenuation β_n at the point scatterer, the scattered replicas $s_{n,k}$ can be written as:

$$\begin{aligned} s_{n,k}(t) &= \beta_n \alpha_k e^{j2\pi f_k (t-\tau'_n)} \\ &= \beta_n s_{1,k}(t) e^{-j2\pi f_k \tau'_n} \end{aligned} \tag{1}$$

for $n > 1$ with c as the speed of light and the propagation delay $\tau'_n = \|\vec{s_n} - \vec{s_1}\| / c$. Based on these definitions, the received signal at the l^{th} gain pattern $g_l(\theta)$ is given as:

$$r_{l,k}(t) = \sum_{n=1}^{N} s_{n,k}(t) g_l(\theta_n) e^{2\pi f_k \tau_n} + n_l(t) \tag{2}$$

where θ_n is the corresponding angle of arrival of the n^{th} source, $\tau_n = \|\vec{s_n} - \vec{e_1}\| / c$ is the propagation delay of the signal from the n^{th} source to the receiver element. Moreover, the $n_l(t)$ represents independent additive Gaussian noise with zero mean and variance σ^2. Without loss of generality, we can drop the term $e^{j2\pi f_k t}$ during the rest of the derivation, since it is common to all signals.

In matrix form, the Equation (2) can be structured with respect to a parametric separation:

$$r_{l,k} = \underbrace{[g_l(\theta_1) \quad g_l(\theta_2) \quad \dots \quad g_l(\theta_N)]}_{\mathbf{A_{DoA}}} \underbrace{\begin{bmatrix} s_{1,k} e^{2\pi f_k \tau_1} \\ s_{2,k} e^{2\pi f_k \tau_2} \\ \vdots \\ s_{N,k} e^{2\pi f_k \tau_N} \end{bmatrix}}_{\mathbf{s_k}} + n_l \tag{3}$$

where $\mathbf{s_k}$ is the source vector that exhibits frequency diversity and $\mathbf{A_{DoA}}$ is the representative matrix of parameter to be estimated.

For unique estimation of the DoA parameter, the number of rows of the matrices $\mathbf{A_{DoA}}$ needs to be at least $N + 1$, together with having unique column vectors for any value of the parameter set of interest [5]. In this fashion, multiple rows on the system of equations can be achieved by stacking the received signals at different gain patterns $g_l(\theta)$. In matrix form, it is given as:

$$\underbrace{\begin{bmatrix} r_{1,k} \\ r_{1,k} \\ \vdots \\ r_{1,k} \end{bmatrix}}_{\mathbf{r_k}} = \underbrace{\begin{bmatrix} g_1(\theta_1) & g_1(\theta_2) & \dots & g_1(\theta_N) \\ g_1(\theta_1) & g_1(\theta_2) & \dots & g_1(\theta_N) \\ \vdots & & & \vdots \\ g_1(\theta_1) & g_1(\theta_2) & \dots & g_1(\theta_N) \end{bmatrix}}_{\mathbf{A_{DoA}}} \underbrace{\begin{bmatrix} s_{1,k} e^{2\pi f_k \tau_1} \\ s_{2,k} e^{2\pi f_k \tau_2} \\ \vdots \\ s_{N,k} e^{2\pi f_k \tau_N} \end{bmatrix}}_{\mathbf{s_k}} + \underbrace{\begin{bmatrix} n_1 \\ n_2 \\ \vdots \\ n_L \end{bmatrix}}_{\mathbf{n}} \tag{4}$$

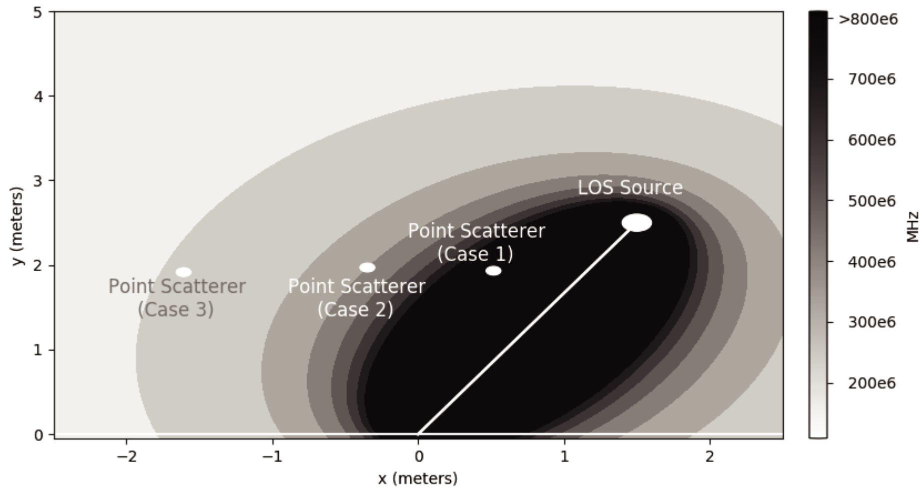

Figure 2: The figure shows the coherence bandwidth as a function of the position of a single point scatterer when there exists a fixed LOS source which is positioned as indicated by the white dot and when the receiver antenna is located at the origin. Three sample cases of the location of the point scatterer are also marked, for reference purposes to the simulation part.

The covariance matrix that is needed by the subspace based estimator is the following:

$$\begin{aligned}\mathbf{R_k} = \mathbb{E}\left\{\mathbf{r_k}\,\mathbf{r_k^H}\right\} &= \mathbf{A_{DoA}}\mathbb{E}\left\{\mathbf{s_k}\,\mathbf{s_k^H}\right\}\mathbf{A_{DoA}^H} + \sigma^2\mathbf{I} \\ &= \mathbf{A_{DoA}}\underbrace{\mathbf{s_k}\,\mathbf{s_k^H}}_{\mathbf{S}_k}\mathbf{A_{DoA}^H} + \sigma^2\mathbf{I} \end{aligned} \tag{5}$$

since the noise is not correlated with the signals and the $\mathbf{s_k}$ is deterministic. The well known correlated source problem is that, subspace based estimators require the $\mathbf{R_k}$ to be full rank [3]. However this is not possible as the $\mathbf{S_k} = \mathbf{s_k}\mathbf{s_k^H}$ is a rank 1 matrix as it is deterministic.

The frequency diversity of the $\mathbf{s_k}$ proposes a smoothing on the covariance matrix $\mathbf{R_k}$, which is represented by the following:

$$\mathbf{R} = \frac{1}{K}\sum_{k=1}^{K}\mathbf{R_k} = \mathbf{A_{DoA}}\left(\frac{1}{K}\sum_{k=1}^{K}\mathbf{s_k}\mathbf{s_k^H}\right)\mathbf{A_{DoA}^H} + \sigma^2\mathbf{I} \tag{6}$$

$$= \mathbf{A_{DoA}}\underbrace{\left(\frac{1}{K}\left[\mathbf{s_1} \quad \mathbf{s_2} \quad \dots \quad \mathbf{s_K}\right]\left[\mathbf{s_1} \quad \mathbf{s_2} \quad \dots \quad \mathbf{s_K}\right]^H\right)}_{\mathbf{S}}\mathbf{A_{DoA}^H} + \sigma^2\mathbf{I} \tag{7}$$

where the smoothed source matrix $\mathbf{S}$ achieves a rank which is equal to the number of sources N, therefore a super-resolution estimator can be applied, as long as the diversity with respect to the frequency exists.

In order to determine the extent of the diversity presented by the scenario of interest, a coherence bandwidth analysis can be made. Assuming that there exists a LOS source together with a single point scatterer, the delay spread of the channel for each location of the point scatterer is given by $\tau_{ds} = \tau_{mp} - \tau_{LOS}$ where τ_{LOS} is the propagation delay of the signal from LOS to the receiver and τ_{mp_max} is the propagation delay of the signal passing through the point scatterer to the receiver. In Figure 2, the coherence bandwidth $B_c \approx 1/\tau_{ds}$ is presented as a function of the location of the single point scatterer in space when a LOS signal is fixed at the indicated position.

The correlated source problem is therefore due to the fact that even if a frequency domain smoothing is applied, it is ineffective against the point scatterers as the impinging signals are assumed to be correlated in terms of frequency if $B_c \gg B_s$ where $B_s = 80\,\text{MHz}$ is the signal bandwidth [6]. Thus, the frequency diversity is not at the same extent for all the possible locations of the point scatterers in the space.

The limitation on the DoA estimation accuracy caused by this fact is examined in the next section, together with the simulation results on the DoA estimation for distinct scatterer positions.

3. SIMULATION RESULTS AND ANALYSIS

The simulations are based on 3 scenarios as shown in Figure 2. The LOS source is fixed and the point scatterer is moved to 3 positions, case 1, 2 and 3. A beam-forming array is placed at the origin which creates 3 gain patterns as shown in Figure 3 that are enough for DoA estimation of 2 sources. The MUSIC estimator is applied using these gain patterns. For each case, the estimator assumes that there exists 2 sources. The impinging DoA of the signal from the LOS source is $\theta_1 = 60^\circ$, distance 2.9 m.

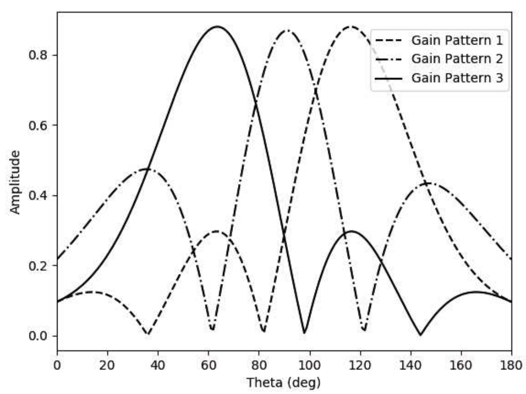

Figure 3: The distinct amplitude gain patterns $g_l(\theta)$ for $l = 1, 2, 3$, presented by the receiver antenna array.

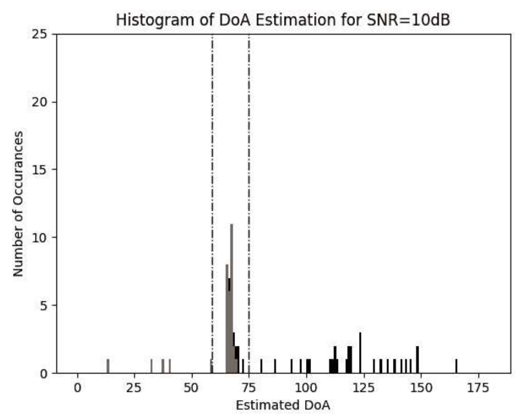

Figure 4: Histogram of the DoA estimation for a LOS source located at a distance of 2.9 m with DoA $\theta_1 = 60^\circ$ and a point scatterer located at a distance of 2 m with DoA $\theta_2 = 75^\circ$. 50 trials have been made with an SNR of 10 dB. The DoA estimations for LOS are represented by gray while the point scatterer estimations are in black color.

In case 1 the DoA of the scatterer is $\theta_2 = 75^\circ$, distance 2 m. The DoA estimations of case 1 are presented in Figure 4. It is clear that the frequency domain decorrelation scheme is not performing well and cannot resolve the 2 separate sources, thus it leads to false estimations as observable in the histogram. This case is regarded as an example, where frequency domain decorrelation is not effective.

In case 2, the point scatterer is located at DoA $\theta_2 = 100^\circ$, distance of 2 m. The DoA estimations for this case are presented in Figure 5. For this case, the sources can be separated, however, the accuracy is poor. This case is regarded as a case, where frequency domain decorrelation is possible, but the accuracy is poor compared to a super-resolution standard.

For case 3, the point scatterer is located at DoA $\theta_2 = 130^\circ$, distance 2.5 m. The DoA estimations for this case are presented in Figure 6. For this last case, the sources can be separated and a super-resolution accuracy is achieved. The coherence bandwidth is relatively small here, therefore it is regarded as a perfectly resolvable case.

From the simulations it is clear that the increased coherence bandwidth of the channel reduces the effectiveness of the frequency domain smoothing regarding decorrelation of the impinging signals. Thus, there exists a relationship between the coherence bandwidth and the MUSIC estimator mean square error with respect to the estimations at a particular SNR. It is represented by Figure 7 for an SNR value of 10 dB.

This measure suggests that for separable sources, a sufficiently low coherence bandwidth of the channel is necessary for effective frequency domain smoothing. For insufficient decorrelation, an assumption of existence of a single source yields a better solution, however, it introduces a bias. This is demonstrated in Figure 8.

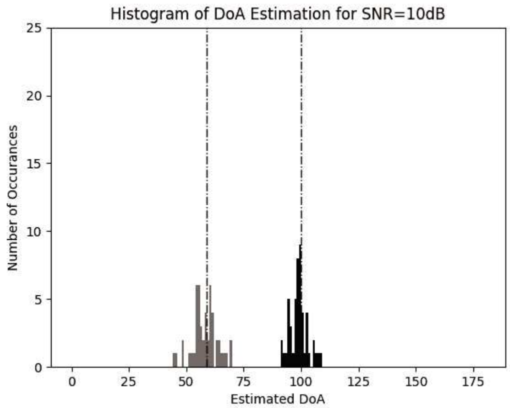

Figure 5: Histogram of the DoA estimation for a LOS source located at a distance of 2.9 m with DoA $\theta_1 = 60°$ and a point scatterer located at a distance of 2 m with DoA $\theta_2 = 100°$. 50 trials have been made with an SNR of 10 dB. The DoA estimations for LOS are represented by gray while the point scatterer estimations are in black color.

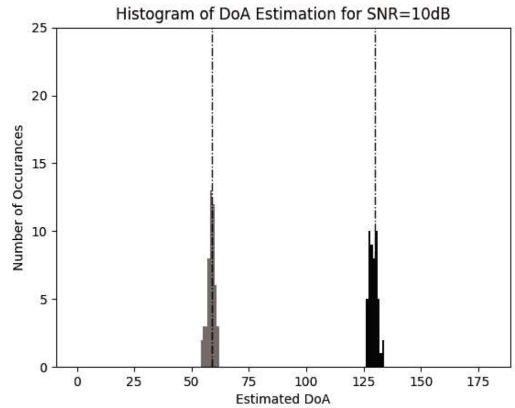

Figure 6: Histogram of the DoA estimation for a LOS source located at a distance of 2.9 m with DoA $\theta_1 = 60°$ and a point scatterer located at a distance of 2.5 m with DoA $\theta_2 = 130°$. 50 trials have been made with an SNR of 10 dB. The DoA estimations for LOS are represented by gray while the point scatterer estimations are in black color.

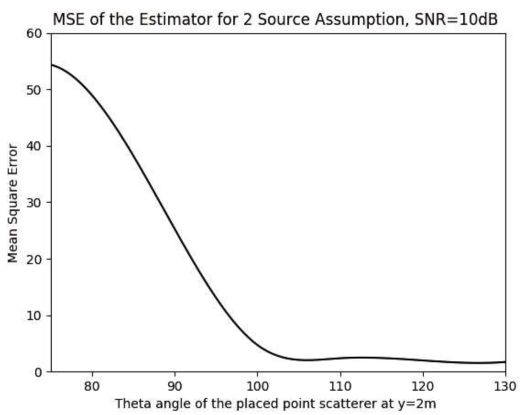

Figure 7: Mean square error of the DoA estimation for a fixed LOS source located at a distance of 2.9 m with DoA $\theta_1 = 60°$, and a single point scatterer that is located at $y = 2$ m and theta angle varied.

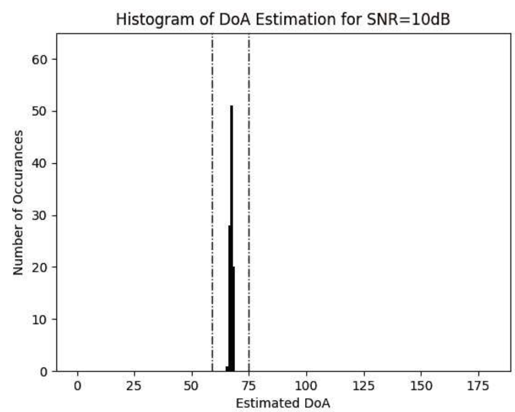

Figure 8: Histogram of the DoA estimation for a LOS source and a point scatterer that has the same properties as Case 1 above, however, a single source is assumed for the MUSIC estimator.

4. CONCLUSION

In this work, a frequency domain decorrelation algorithm is presented together with its limitations, referring to the simulation results. Subspace based DoA estimation algorithms achieves super-resolution, as long as the impinging signals from the sources are not correlated. However, for the scenarios where there exists multipath propagation, the scattered signals are correlated and it is destructive on the estimator performance, so that a decorrelation is necessary. It is demonstrated that frequency domain smoothing for the decorrelation of the signals is only effective when the channel coherence bandwidth is sufficiently small, with respect to the signal bandwidth. This is the limitation of the frequency domain smoothing and is supported by three distinct case simulations in this work. Lastly, a demonstration on the relation between the mean square error of MUSIC estimator and the scatterer position for a fixed LOS source, is presented. For the case where the coherence bandwidth is sufficiently small, the frequency domain smoothing appears to be a very good option for decorrelating the LOS source signal from the scattered signals due to the presence of the point scatterers.

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